ICMI Study 23

UNIVERSIDADE DE MACAU
UNIVERSITY OF MACAU

Primary Mathematics Study on Whole Numbers

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It is time to reveal what students with MLD know, rather than what they do not know

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Knowing what students know

is *the* starting point of good instruction

Looking for what students know

- implies a change in assessment:
  Assessing (MLD) students' mathematical potential

- implies a change in teaching:
  Building on what (MLD) students already know

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Examples

IMPULSE project

1. Offering students problems in which they can show their competence
   - 1a. Within curriculum: Subtraction problems
   - 1b. Beyond curriculum: Combinatorial problems

FaSMEd project

2. Offering students optional auxiliary tools: Percentage problems

***"Going across the grain"* study** by Watson (2002)

3. Advanced mathematical thinking by low attaining students

Example 1a

Offering students problems by which they can show their competence

Subtraction problems that elicit strategies

IMPULSE

Example 1a

Study with Special Education students

56 students from 14 classes in SE school
8-12 years old, mathematics level Grade 2
15 problems

Students who are weak in mathematics should be taught just ***one procedure****: Subtraction should be solved by Direct Subtraction and not by Indirect Addition*

Is there evidence for this?

IMPULSE

Example 1a

62 euro

29 euro discount

1 2 3 4 5 6 7 8 9 0

answer:

Direct Subtraction
DS strategy

Taking Away
Context

next

Example 1a

space for 51 cards

49 are already included

1 2 3 4 5 6 7 8 9 0

answer:

Indirect addition
IA strategy

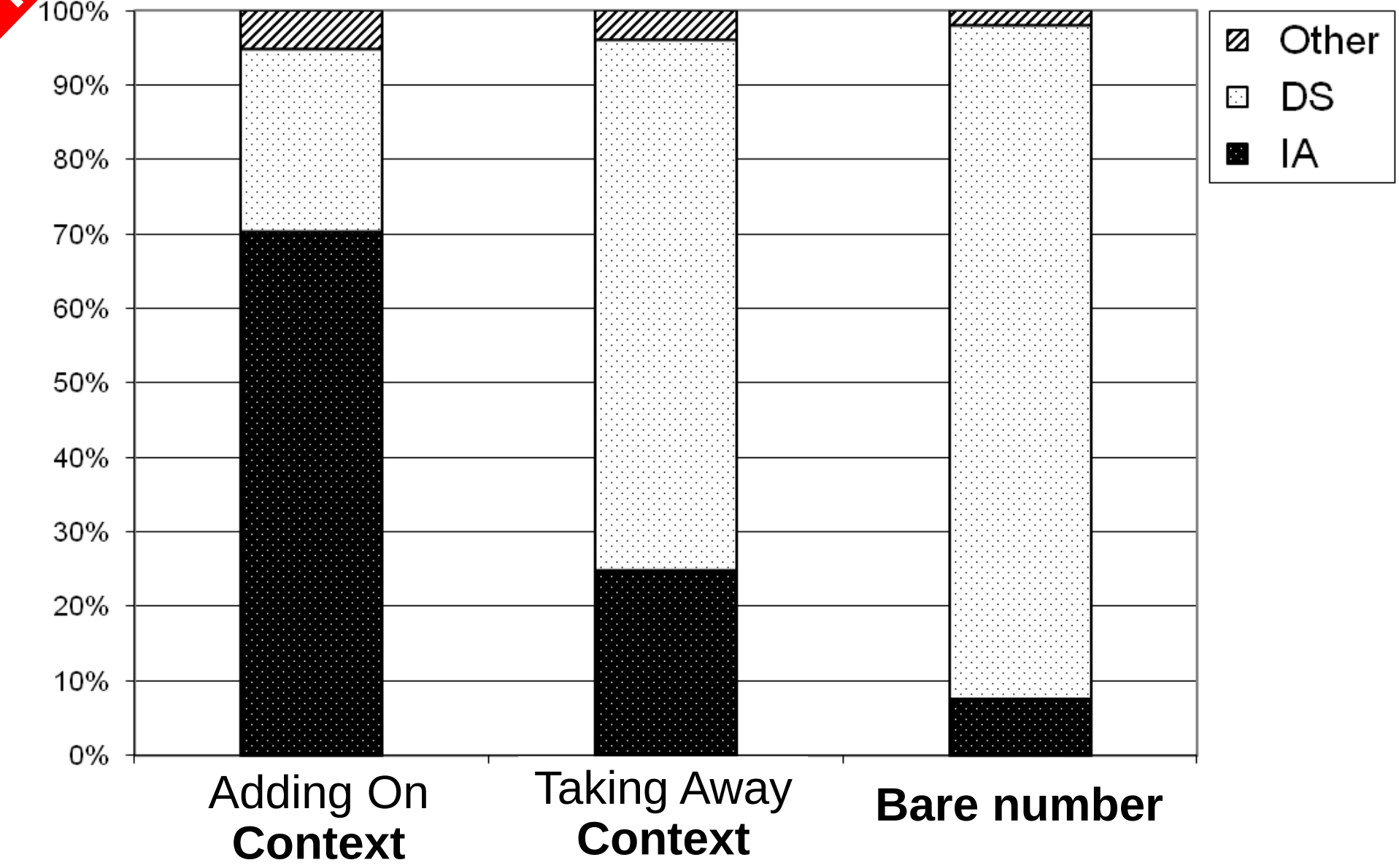
Adding On
Context

next

IMPULSE

Example 1a

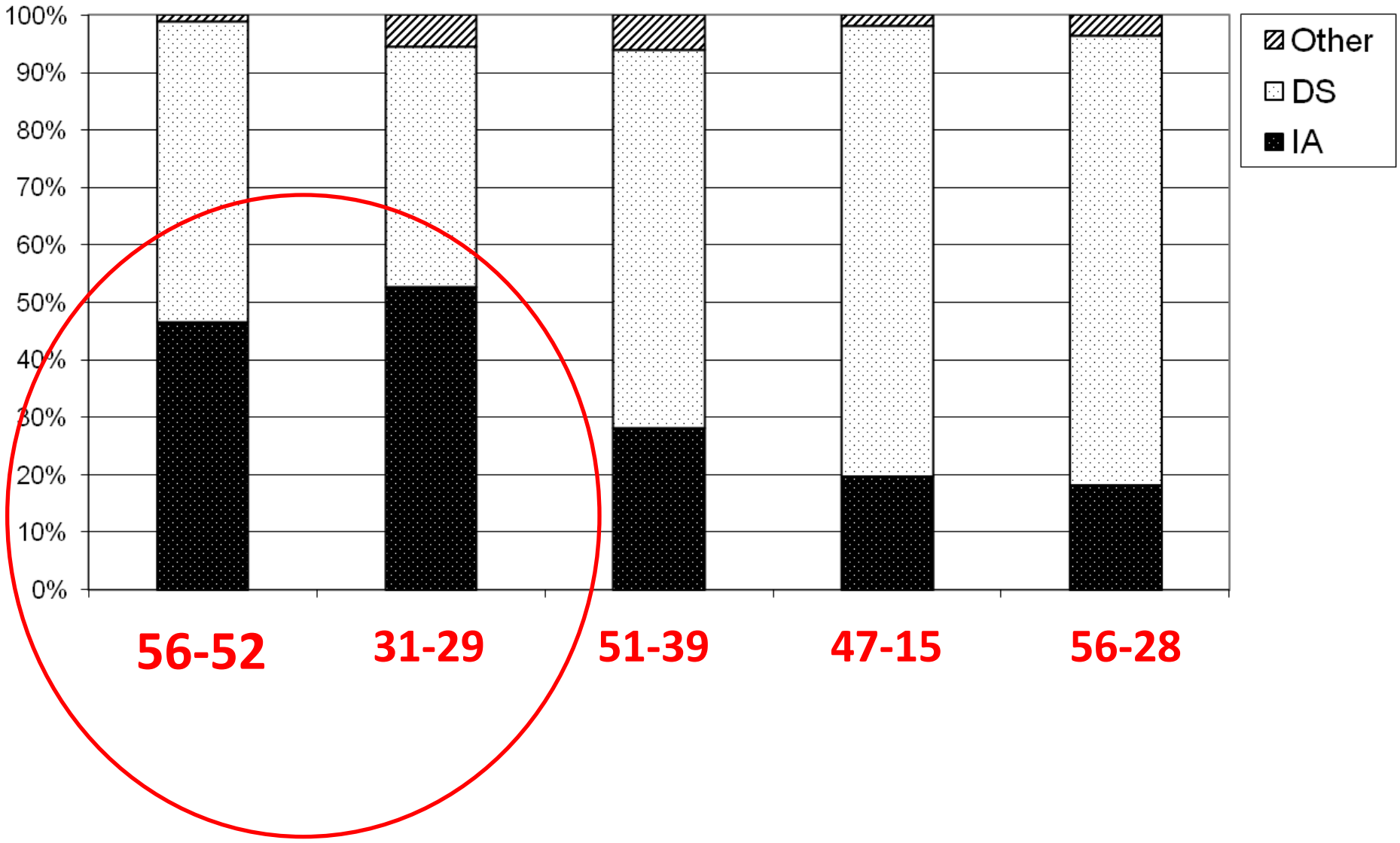
IA use and problem format



IMPULSE

Example 2a

IA use and numbers involved



Example 1a

Conclusions

- SE students can make spontaneous use of IA
  - DS 63%
  - IA 34%
  - Average IA use per student 4.6 (min 0, max 8)
- SE students are rather flexible in applying IA
- SE students are quite successful when applying IA
  - DS 51% correct
  - IA 68% correct

Example 1b
Offering students problems in which they can show their competence:
Combinatorial problems

IMPULSE

Example 1b

Research question:

Can special education students solve combinatorial problems?

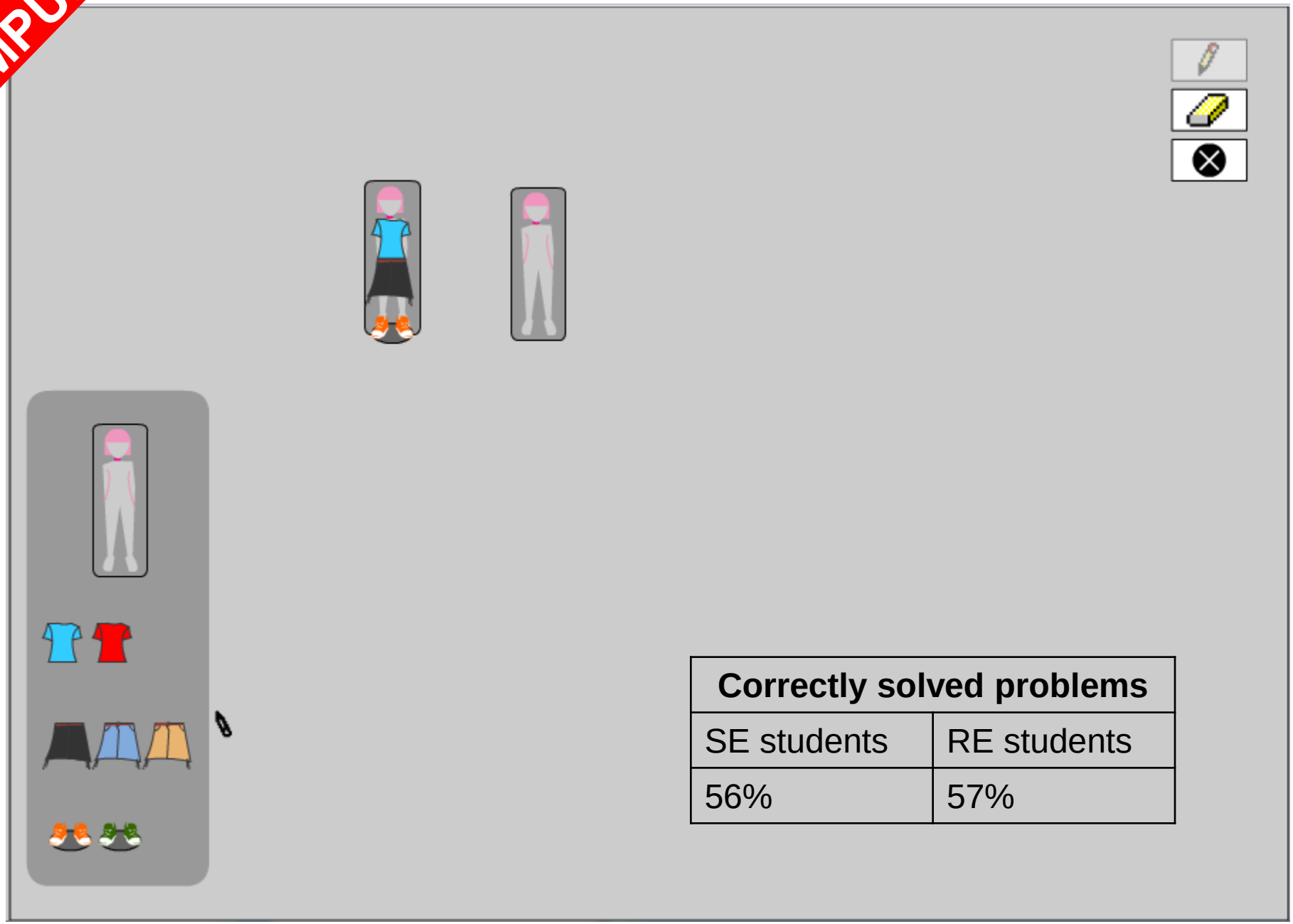
Participants:

84 students (age *M* = 11.1) from 5 SE schools
76 students (age *M* = 9.4) from 5 RE schools
mathematics levels Grade 2-5

Instrument:

6 combinatorial problems in ICT environment

Example 1b

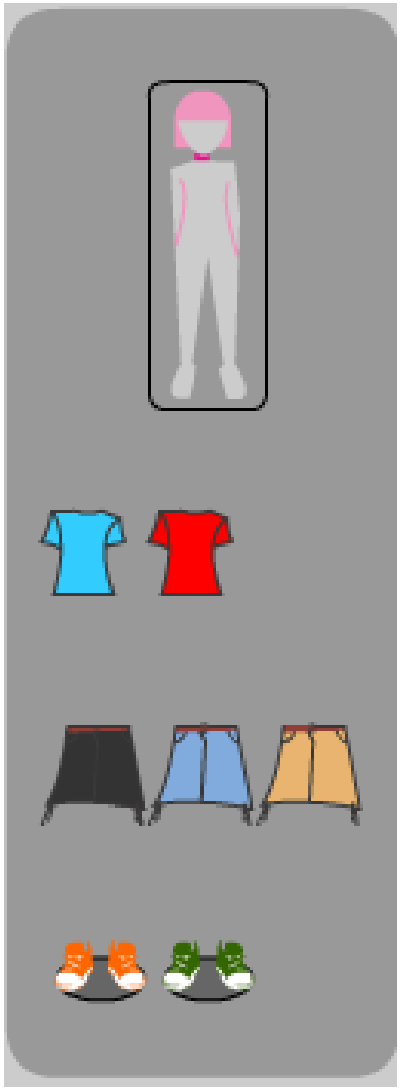


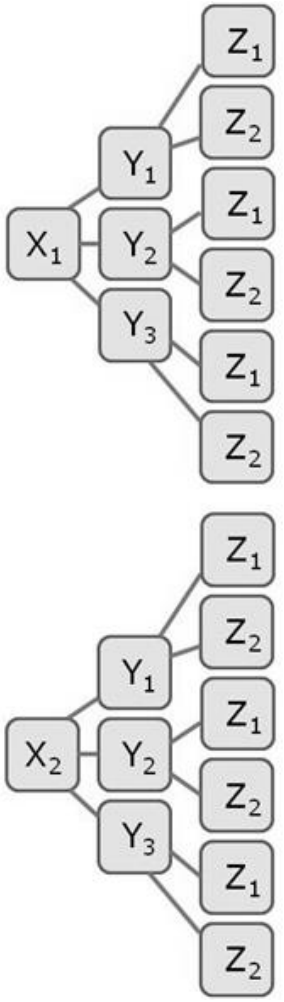
Correctly solved problems	
SE students	RE students
56%	57%

Example 1b

Strategy use:

- systematic
- semi-systematic
- non-systematic

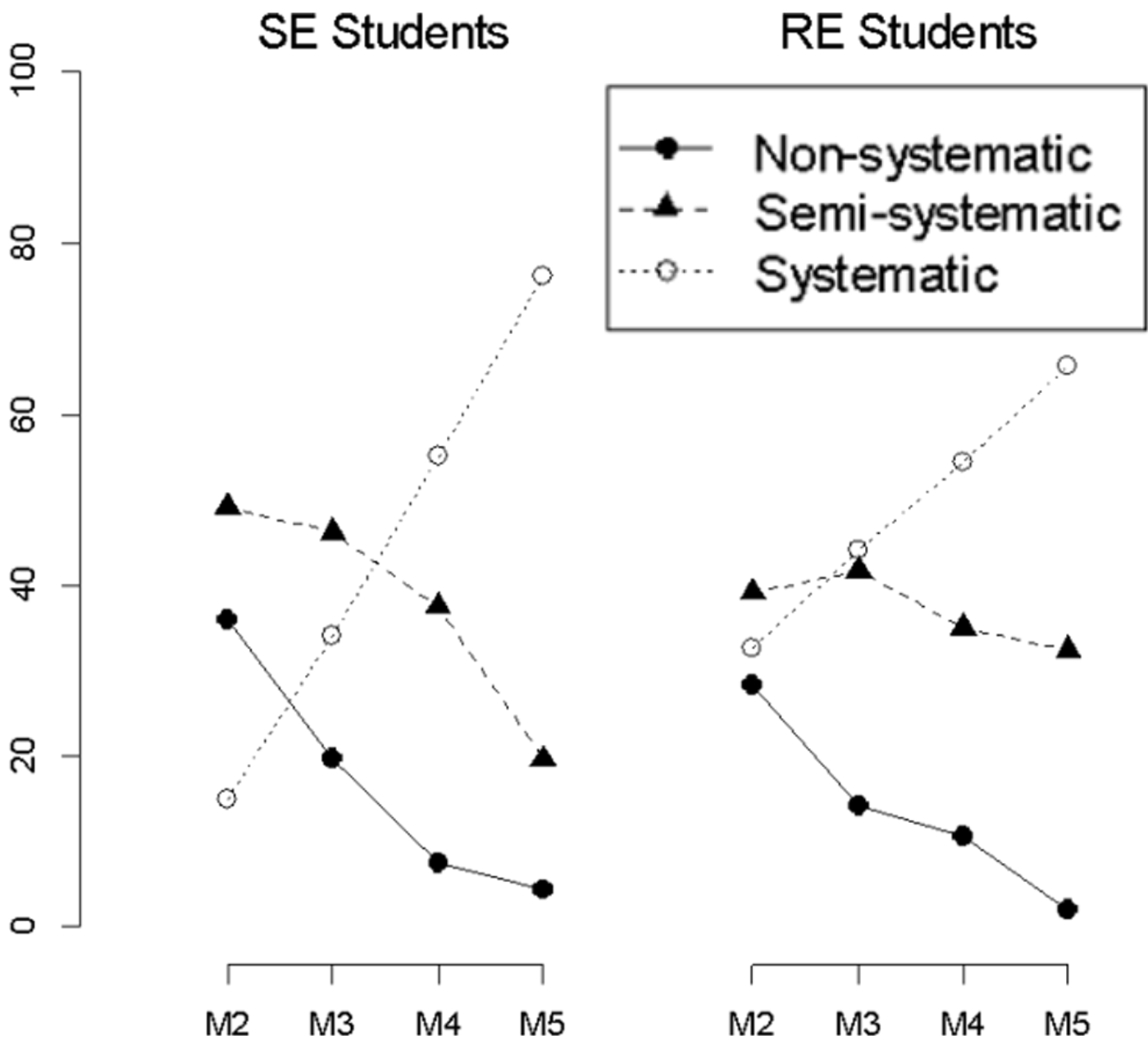




IMPULSE

Example 1b

Frequency (%) of strategy use



Example 2
Offering students optional auxiliary tools:
Percentage problems

FaSMEd

Example 2

Digital Assessment Environment

- Web-based
- Monitoring function
- Problems based on key competencies
- **Auxiliary tools**

Six problems on percentage

Grade 6 teacher about his student Ducan:
“He belongs to the low-level stream in my class and now he did three of the six problems correctly!”

Example 2

Problem 1

When a battery is full, it will work 120 hours.
It is still charged for 40%.
For how many hours will this battery still work?
Answer: … hours

FaSMEd

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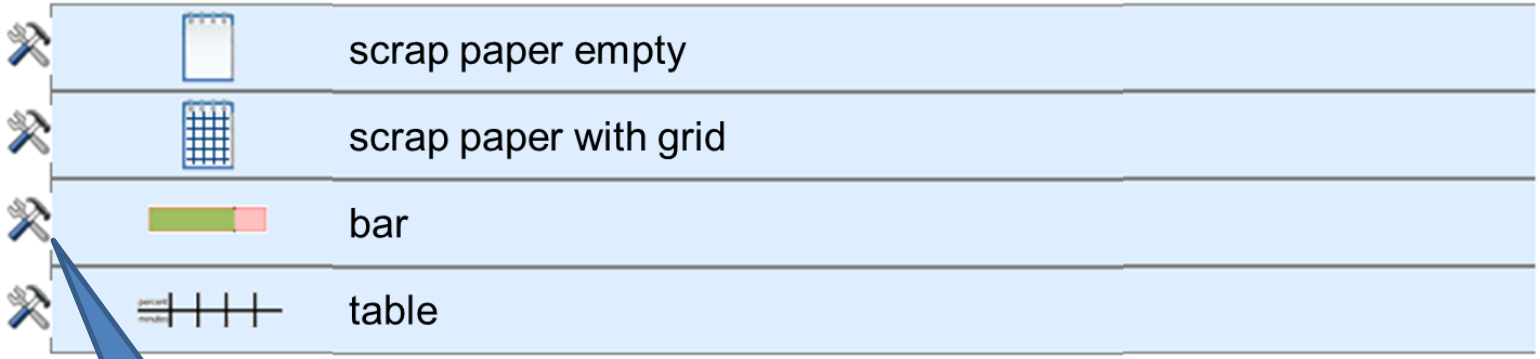

The tool icon is clicked

FaSMEd

Example 2

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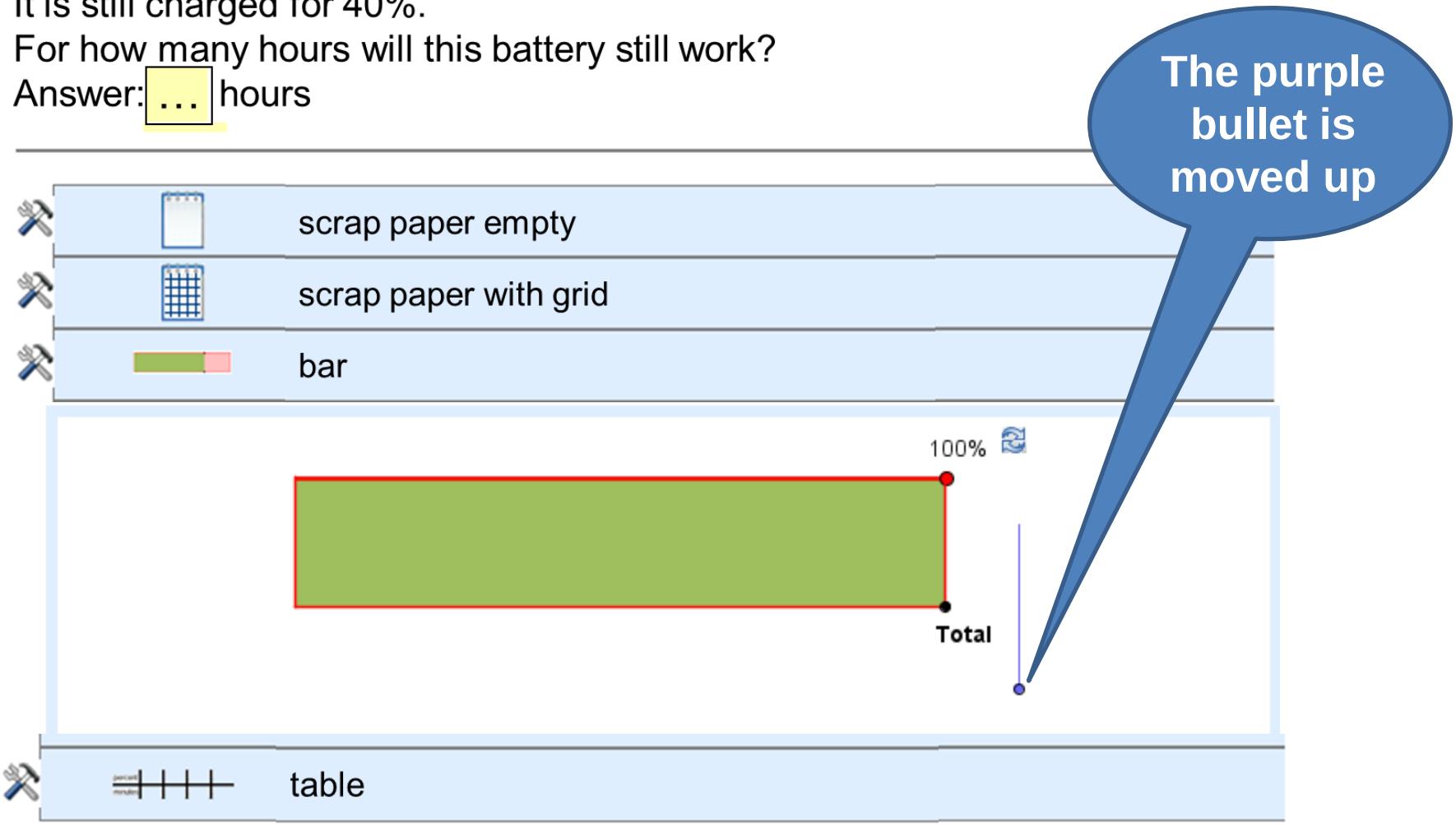
Answer: ... hours



Example 2

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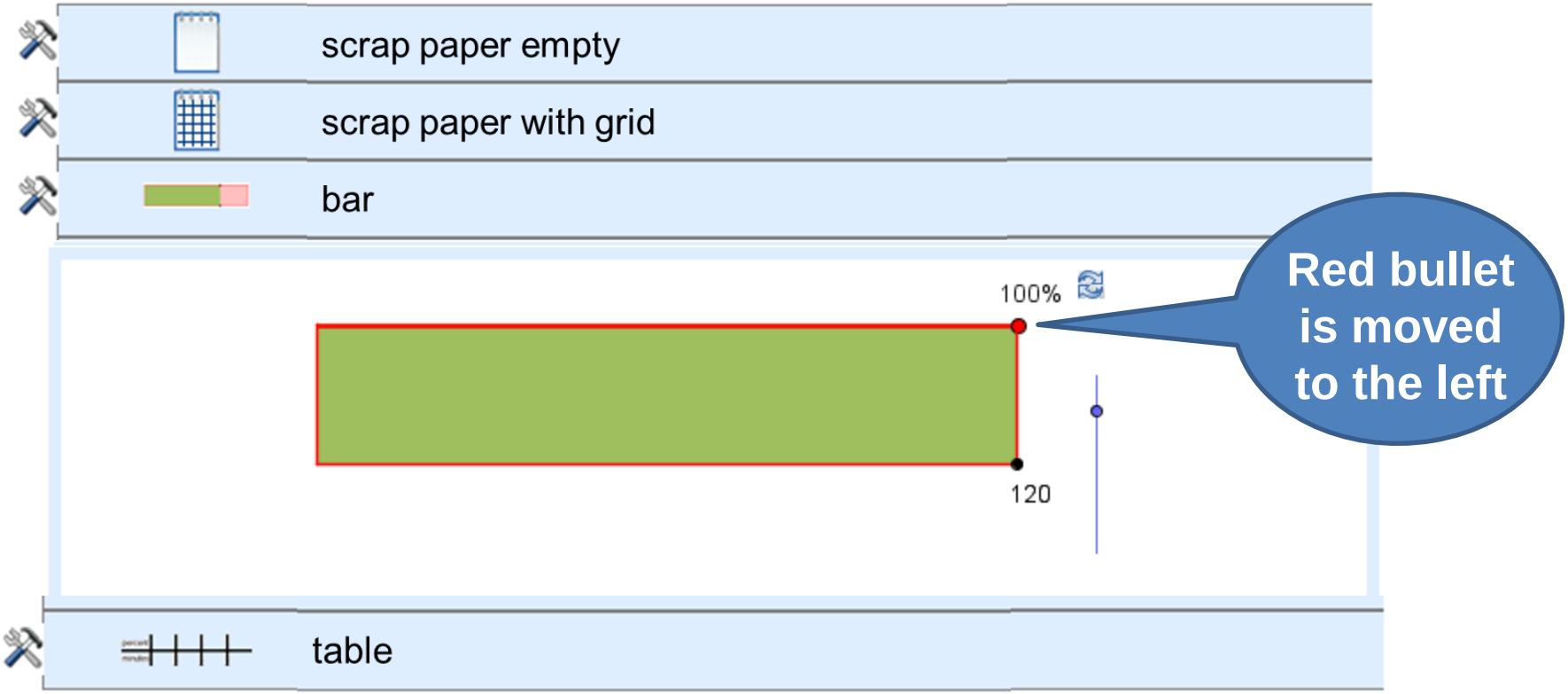


FaSMEd

Example 2

Problem 1

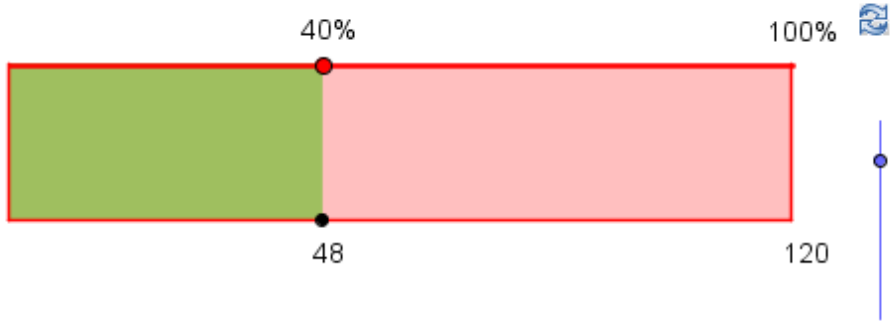
When a battery is full, it will work 120 hours.
It is still charged for 40%.
For how many hours will this battery still work?
Answer: ... hours

The answer is filled in

scrap paper empty

scrap paper with grid

bar

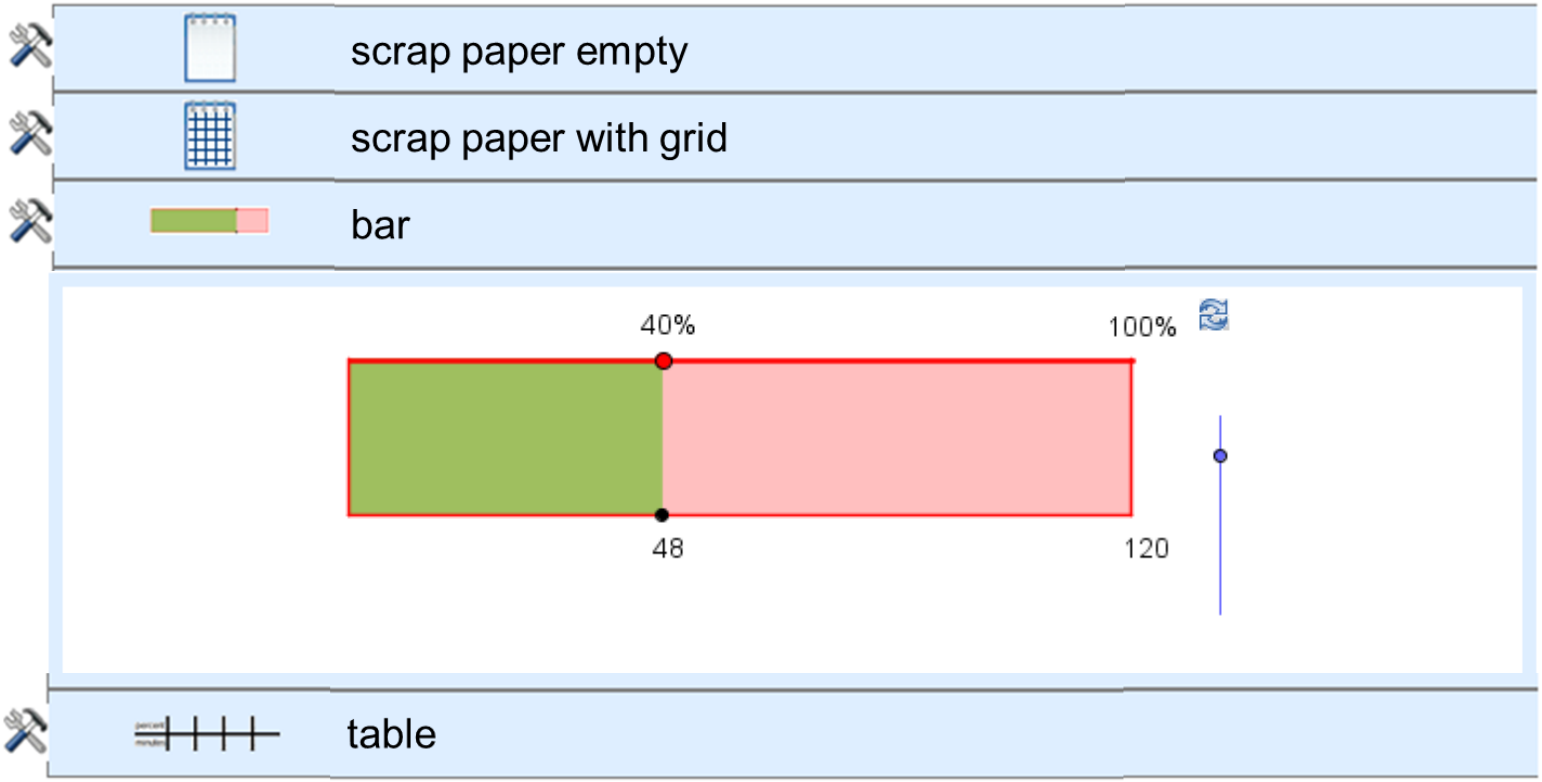


table

Example 2

Problem 1

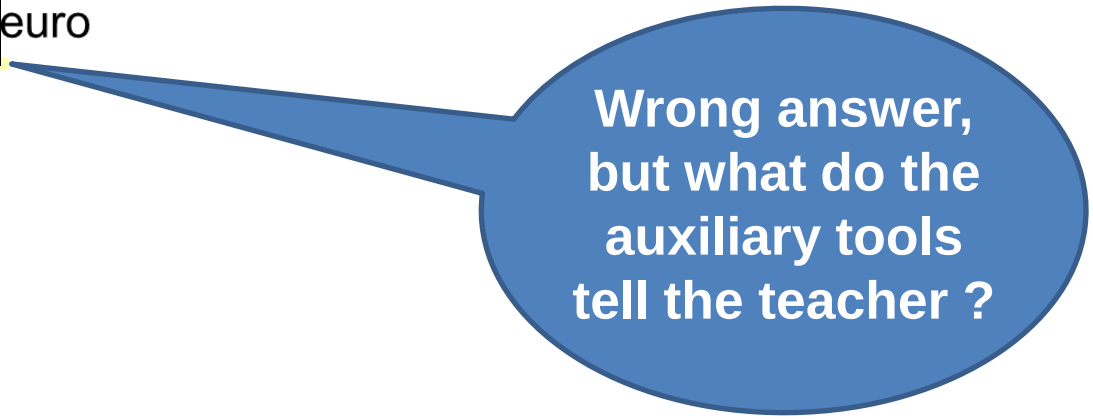
When a battery is full, it will work 120 hours.
It is still charged for 40%.
For how many hours will this battery still work?
Answer: 48 hours



Example 2

Problem 2

A cell phone costs 70 euro.
You get a discount of 20%.
What do you have to pay?
Answer 66 euro



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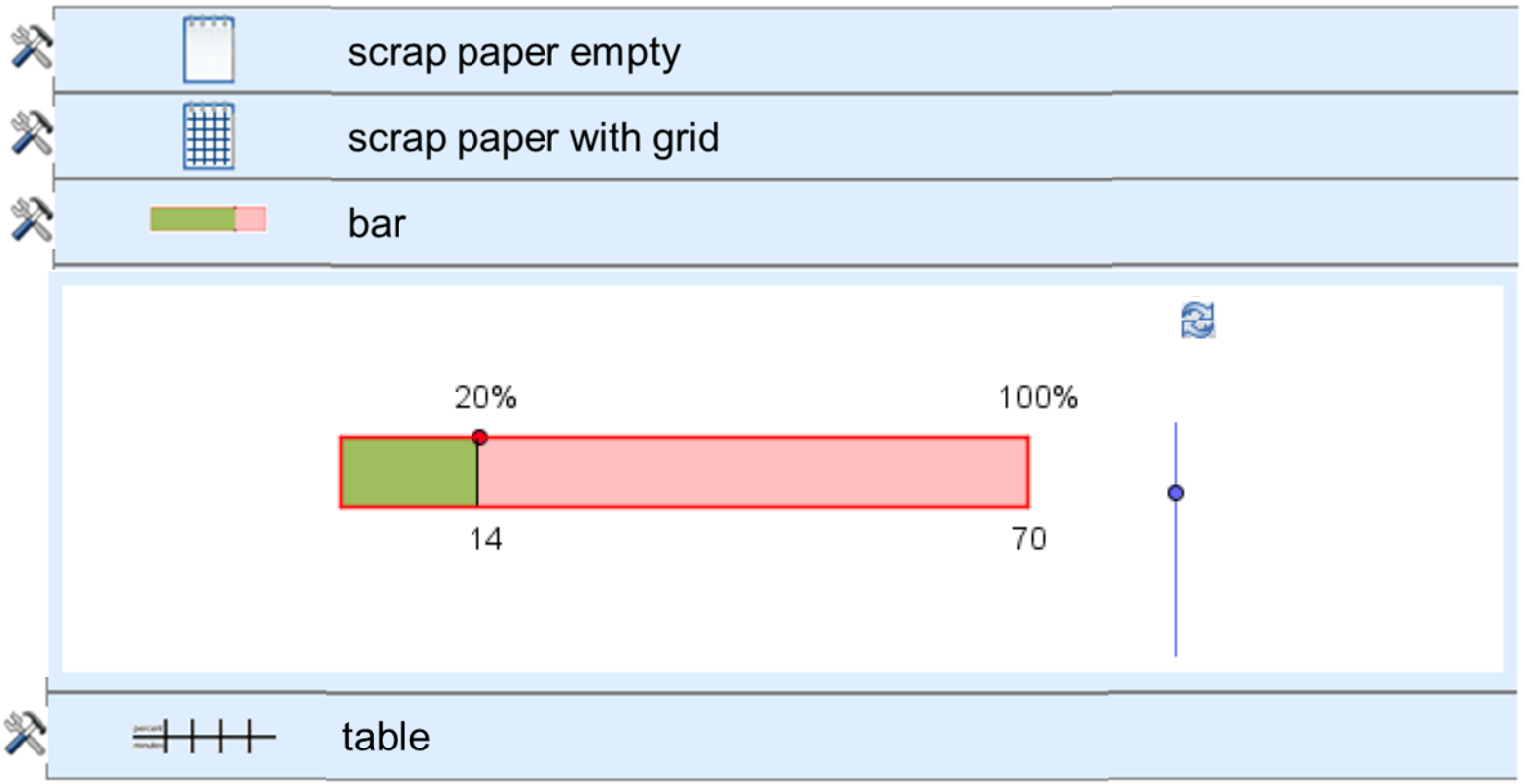

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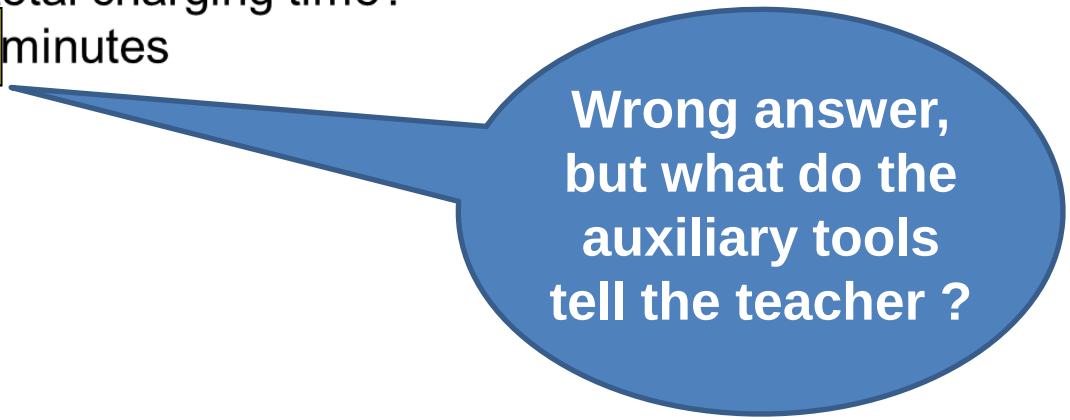
Example 2

Problem 5

In 24 minutes the battery is charged for 75%.
What is the total charging time?
Answer: 30 minutes



Example 2

Problem 5

In 24 minutes the battery is charged for 75%.
What is the total charging time?
Answer: 30 minutes

- scrap paper empty
- scrap paper with grid
- bar
- table

75	50	25	...	...	...	...	...
24	12	6	...	...	...	...	...

Example 3
Advanced mathematical thinking by low attaining students

Watson

Example 3

Pergamon

Journal of Mathematical Behavior
20 (2002) 461–475

Mathematical Behavior

Instances of mathematical thinking among low attaining students in an ordinary secondary classroom

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Abstract

This paper is a report of a classroom research project whose aim was to find out whether low attaining 14-year-old students of mathematics would be able to think mathematically at a level higher than recall and reproduction during their ordinary classroom mathematics activities. Analysis of classroom interactive episodes revealed many instances of mathematical thinking of a kind which was not normally exploited, required or expected in their classes. Five episodes are described, comparing the students' thinking to that usually described as "advanced." In particular, some episodes suggest the power of a type of prompt which can be generalized as "going across the grain."

Watson

Example 3

"Going across the grain" study by Watson (2002)

" 'Low attaining students' are generally classified […] on the basis of accumulated incompetence in tests and other written work."

~~***Deficiency-based approach***~~

Proficiency-based approach

Watson

Example 3

$5 \times 7 =$	$7 \times 5 =$	$35/5 =$	$35/7 =$
$6 \times 7 =$	$7 \times 6 =$	$42/6 =$	$42/7 =$
$7 \times 7 =$	–	$49/7 =$	–
$8 \times 7 =$	$7 \times 8 =$	$56/8 =$	$56/7 =$

"Students soon realized that the answers to the 1st and 2nd column were obtained by adding 7. …"

Then they were asked to continue the horizontal pattern, which was completely new for them

$23 \times 7 = 161$ → $7 \times 23 = 161$ → $161/23 = 7$ → $161/7 = 23$

"All could do this after some thought"

These low attainers showed that they can
- identify and use patterns
- work with abstractions and relations

Research on MLD needs a
proficiency-based approach

It is time to reveal
what students with MLD know,
rather than what they do not know

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